Technical Notes

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View Factors between Disk/Rectangle and Rectangle in Parallel and Perpendicular Planes

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Nomenclature

A_1, A_2	=	areas of the two finite surfaces
B, L, S	=	dimensions of rectangle and square
C_1, C_2	=	contours of finite areas A_1 and A_2, respectively
D	=	distance between disk and rectangle, as in Fig. 1
$F_{1\text{–}2}$	=	view factor between finite areas A_1 and A_2
R	=	radius of the disk
x, y, z	=	Cartesian coordinates
α,	=	angles between squares, as in Fig. 2, radians
θ	=	angular coordinate measured from x axis on x–y plane, in anticlockwise direction, radians

Subscript

i	=	corresponding to ith path

Introduction

ANALYSIS of radiation heat transfer requires expressions or data for view factors between finite surfaces. A review of the literature shows that a comprehensive catalog of view factors [1] provides expressions or data for several differential and finite geometrical configurations; closed form solutions have been presented by Ehlert and Smith [2] for view factors between two rectangles when they are perpendicular to each other and all the edges are either parallel or perpendicular. The catalog of view factors [1] also shows that expressions or data for several pairs of finite surfaces are not available, which are quite common in many applications.

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Hence, presently, three configurations of finite surfaces have been considered: the view factor between 1) a disk and a rectangle on parallel planes (as shown in Fig. 1); 2) a disk and a rectangle in perpendicular planes, when the disk is placed on the plane containing the rectangle with its edges either parallel or perpendicular to the plane of the disk (as shown in Fig. 1); 3) rectangles in perpendicular planes, when an edge of one rectangle is placed on the plane containing the other rectangle, which is placed arbitrarily (as shown in Fig. 2). Closed form solutions have been developed to the extent possible, and correlations have been presented for three specific cases, after evaluating the nonintegrable terms numerically.

Analysis

Following Sparrow [3], the view factor between two finite surfaces can be written as

$$F_{1-2} = \frac{1}{4\pi A_1} \oint_{C_1} \oint_{C_2} \ell n[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2](\mathrm{d}x_1\,\mathrm{d}x_2 + \mathrm{d}y_1\,\mathrm{d}y_2 + \mathrm{d}z_1\,\mathrm{d}z_2) \tag{1}$$

where C_1 and C_2 are the contours of surfaces, which will be subdivided along the paths constituting the contours.

Case 1 View factor between a disk and a rectangle in parallel planes: Consider the view factor between a disk and a rectangle in parallel planes, as shown in Fig. 1. The coordinate system is defined such that the disk lies on the x–y plane having origin at its center and the edges of the rectangle are either parallel or perpendicular to the x and y axes. The corner points of the rectangle are denoted as $P_{2,i}$, along the direction of integration. Since $z_1 = 0$ and $z_2 = D$, integrations along $\mathrm{d}z_1\,\mathrm{d}z_2$ vanish. Thus Eq. (1) reduces to

$$F_{1-2} = \frac{1}{4\pi A_1} \oint_{C_1} \oint_{C_2} \ell n[(x_2 - x_1)^2 + (y_2 - y_1)^2 + D^2] \times (\mathrm{d}x_1\,\mathrm{d}x_2 + \mathrm{d}y_1\,\mathrm{d}y_2) \tag{2}$$

On contour C_2, y_2 remains constant along the paths $P_{2,1}$–$P_{2,2}$ and $P_{2,3}$–$P_{2,4}$, while x_2 varies and hence the integrations along $\mathrm{d}y_1\,\mathrm{d}y_2$ vanish. Further, x_2 remains constant along the paths $P_{2,2}$–$P_{2,3}$ and $P_{2,4}$–$P_{2,1}$, while y_2 varies and hence integrations along $\mathrm{d}x_1\,\mathrm{d}x_2$ vanish. On contour C_1, x_1 and y_1 are represented in polar form and the closed form solution (to the possible extent) of Eq. (2) along the said paths can be shown to be

$$F_{1-2} = \frac{1}{4\pi A_1}\left[\sum_{i=1,3}^{2,4}\sum_{j=1}^{2}(-1)^j G_1(R,\theta,x_{2,i},y_2) + \sum_{i=2,4}^{3,1}\sum_{j=1}^{2}(-1)^j G_2(R,\theta,x_2,y_{2,i})\right] \tag{3a}$$

where

$$
\begin{aligned}
G_1(R,\theta,x_{2,i},y_2) = {} & R^2\Bigg(\frac{1}{2\left(b_1^2+c_1^2\right)^2}\bigg\{b_1c_1\theta\left(-2a_1^2+b_1^2+c_1^2\right) \\
& + 4a_1b_1c_1\sqrt{a_1^2-b_1^2-c_1^2}\tan^{-1}\left[\frac{c_1+(a_1-b_1)\tan(\theta/2)}{\sqrt{a_1^2-b_1^2-c_1^2}}\right]\bigg\} \\
& + \frac{a_1}{2}\frac{c_1\sin\theta-b_1\cos\theta}{b_1^2+c_1^2}+\frac{\cos 2\theta}{8} \\
& + \left\{\frac{\left[b_1^2\left(2a_1^2-b_1^2\right)+c_1^2\left(c_1^2-2a_1^2\right)\right]}{4\left(b_1^2+c_1^2\right)^2}-\frac{1}{4}\cos 2\theta\right\} \\
& \times \ln(a_1+b_1\cos\theta+c_1\sin\theta)\Bigg)-\frac{Rx_{2,i}}{b_1^2+c_1^2} \\
& \times\left\{-2c_1\sqrt{a_1^2-b_1^2-c_1^2}\tan^{-1}\left[\frac{c_1+(a_1-b_1)\tan(\theta/2)}{\sqrt{a_1^2-b_1^2-c_1^2}}\right]\right. \\
& + a_1c_1\theta-\left[a_1b_1+\left(b_1^2+c_1^2\right)\cos\theta\right]\ln(a_1+b_1\cos\theta \\
& \left. + c_1\sin\theta)+3\left(b_1^2+c_1^2\right)\cos\theta\right\} \\
& - 2R\int_0^{2\pi}\sin\theta\sqrt{(y_2+R\sin\theta)^2+D^2}\tan^{-1} \\
& \times\left[\frac{x_{2,i}-R\cos\theta}{\sqrt{(y_2+R\sin\theta)^2+D^2}}\right]\mathrm{d}\theta
\end{aligned}
\tag{3b}
$$

in which $a_1 = x_{2,i}^2+y_2^2+D^2+R^2$; $b_1=-2Rx_{2,i}$; and $c_1=-2Ry_2$; and

$$
\begin{aligned}
G_2(R,\theta,x_2,y_{2,i}) = {} & Ry_{2,i}\Bigg\{-2b_2\sqrt{a_2^2-b_2^2-c_2^2}\bigg(\frac{Ra_2c_2}{y_{2,i}}+b_2^2 \\
& + c_2^2\bigg)\tan^{-1}\left[\frac{c_2+(a_2-b_2)\tan(\theta/2)}{\sqrt{a_2^2-b_2^2-c_2^2}}\right] \\
& + \left[\frac{R\left(b_2^2-c_2^2\right)\left(b_2^2+c_2^2-2a_2^2\right)}{y_{2,i}(b_2^2+c_2^2)^2}+\sin\theta+\frac{R\cos\theta}{4y_{2,i}}+\frac{a_2c_2}{b_2^2+c_2^2}\right] \\
& \times\ln(a_2+b_2\cos\theta+c_2\sin\theta)-\frac{b_2c_2R\theta\left(-2a_2^2+b_2^2+c_2^2\right)}{2y_{2,i}\left(b_2^2+c_2^2\right)^2} \\
& - \frac{R\cos 2\theta}{8y_{2,i}}-\sin\theta+\frac{a_2}{2y_{2,i}}\frac{[b_2\theta y_{2,i}-R(c_2\sin\theta-b_2\cos\theta)]}{b_2^2+c_2^2}\Bigg\} \\
& - 3R\sin\theta y_{2,i}+2R\int_0^{2\pi}\cos\theta\sqrt{(x_2-R\cos\theta)^2+D^2}\tan^{-1} \\
& \times\left[\frac{y_{2,i}-r\sin\theta}{\sqrt{(x_2-R\cos\theta)^2+D^2}}\right]\mathrm{d}\theta
\end{aligned}
\tag{3c}
$$

in which $a_2=x_2^2+y_{2,i}^2+D^2+R^2$; $b_2=-2Rx_2$; $c_2=-2Ry_{2,i}$.The lower and upper limits of integration on contour C_1 are $\theta=0$ and $\theta=2\pi$, respectively. On contour C_2, $j=1$, 2 denotes lower and upper limits of integrations, respectively, which corresponds to endpoint coordinates.

Alternatively, correlations are developed for the specific case of a view factor between a disk of unit radius and a rectangle of size $L\times B$ which are placed coaxially, after evaluating the last terms in Eqs. (3b) and (3c) using Simpson's one-third rule. The contour of the disk was discretized using 10^5 divisions, and four different correlations are presented to minimize the maximum error in the predicted view factor: the correlation for the range $0.1\le L\le 2.0$, $0.1\le B\le 2.0$, and $0.1\le D\le 10.0$,

$$
F_{2\text{–}1}=\frac{1.0152(1+L^{1.0251})(1+B^{3.4915})}{(1+L^{1.195})(1+B^{3.656})[D^{1.9767}+(1+D)^{0.3047}-0.0175]}
\tag{4a}
$$

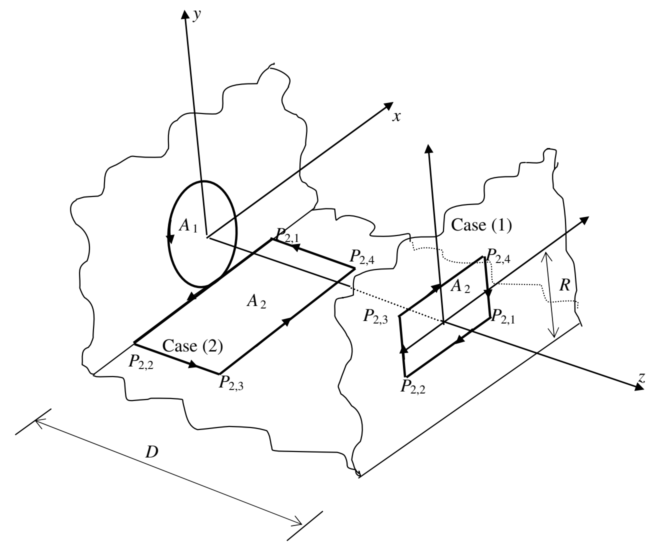

Fig. 1 Schematic diagram for cases 1 and 2.

is based on 14,544 data points. The maximum error is $\pm 12.03\%$ and the correlation coefficient is 0.999. The correlation for the range $2.0\le L\le 10.0$, $2.0\le B\le 10.0$, and $0.1\le D\le 2.0$,

$$
F_{2\text{–}1}=\frac{3.2718(1+D^{1.6491})^{0.2834}}{(D^{1.5138}+L^{0.495}B^{0.495})^{2.0417}}
\tag{4b}
$$

is based on 1280 data points. The maximum error is $\pm 12.59\%$ and the correlation coefficient is 0.997. The correlation for the range $2.0\le L\le 10.0$, $2.0\le B\le 10.0$, and $2.0\le D\le 10.0$,

$$
F_{2\text{–}1}=\frac{1.1947(1+L^{0.4609})(1+B^{0.46})}{(1+L^{0.5111})(1+B^{0.5102})(D^{2.0405}+B^{1.195}+L^{1.1949}-3.4734)^{2.0405}}
\tag{4c}
$$

is based on 1024 data points. The maximum error is $\pm 11.42\%$ and the correlation coefficient is 0.998. The correlation for the range $0.1\le L\le 2.0$, $2.0\le B\le 10.0$, and $0.1\le D\le 10.0$,

$$
\begin{aligned}
F_{2\text{–}1} = {} & 3.0932D^{-0.1128}(A_2+A_2^2+D+D^2+D^4)^{-0.0188}(D^{1.0657} \\
& + L^{0.0215}B^{0.4893})^{-2.2497}
\end{aligned}
\tag{4d}
$$

is based on 23,040 data points. The maximum error is $\pm 18.64\%$ and the correlation coefficient is 0.996.

The view factor between a disk and a rectangle placed arbitrarily on a parallel plane can be calculated using the correlations presented and view factor algebra.

Case 2 View factor between a disk and a rectangle in perpendicular planes: Consider the view factor between a disk and a rectangle in perpendicular planes, as shown in Fig. 1, when the disk is placed on the plane containing a rectangle, whose edges are either parallel or perpendicular to the plane of the disk. The coordinate system is defined such that the disk lies on the x–y plane with the origin at its center and the edges of the rectangle being parallel or perpendicular to the x and z axes. The corner points of the rectangle are denoted as $P_{2,i}$, along the direction of integration. Since $z_1=0$ and $y_2=-R$, integrations along $\mathrm{d}z_1\,\mathrm{d}z_2$ and $\mathrm{d}y_1\,\mathrm{d}y_2$ vanish. Thus Eq. (1) reduces to

$$
F_{1\text{–}2}=\frac{1}{4\pi A_1}\oint_{C_1}\oint_{C_2}\ln\left[(x_2-x_1)^2+(R+y_1)^2+z_2^2\right]\mathrm{d}x_1\,\mathrm{d}x_2
\tag{5}
$$

On contour C_2, x_2 remains constant along the paths $P_{2,2}$–$P_{2,3}$ and $P_{2,4}$–$P_{2,1}$ and hence the integrals along $\mathrm{d}x_1\,\mathrm{d}x_2$ vanish. Further, z_2 remains constant along the paths $P_{2,1}$–$P_{2,2}$ and $P_{2,3}$–$P_{2,4}$, while x_2 varies. On contour C_1, x_1 and y_1 are represented in polar form and the closed form solution (to the possible extent) of Eq. (5), along the said paths can be shown to be

$$F_{1\text{–}2}=\frac{1}{4\pi A_1}\sum_{i=1,3}^{2,4}\sum_{j=1}^{2}(-1)^j G(R,\theta,x_{2,i},z_2) \tag{6a}$$

where

$$\begin{aligned}
G(R,\theta,x_{2,i},z_2)={}&R^2\left(\frac{bc}{2(b^2+c^2)^2}\left\{\theta(b^2+c^2-2a^2)\right.\right.\\
&\left.+4a\sqrt{a^2-b^2-c^2}\tan^{-1}\left[\frac{c+(a-b)\tan(\theta/2)}{\sqrt{a^2-b^2-c^2}}\right]\right\}\\
&+\frac{ac\sin\theta-b\cos\theta}{2\quad b^2+c^2}+\frac{\cos2\theta}{8}-\frac{x_{2,i}}{R(b^2+c^2)}\\
&\times\left\{ac\theta-2c\sqrt{a^2-b^2-c^2}\tan^{-1}\left[\frac{c+(a-b)\tan(\theta/2)}{\sqrt{a^2-b^2-c^2}}\right]\right.\\
&-[ab+(b^2+c^2)\cos\theta]\ln(a+b\cos\theta+c\sin\theta)\\
&\left.+3(b^2+c^2)\cos\theta\right\}+\left\{\frac{[b^2(2a^2-b^2)+c^2(c^2-2a^2)]}{4(b^2+c^2)^2}\right.\\
&\left.\left.-\frac{\cos2\theta}{4}\right\}\ln(a+b\cos\theta+c\sin\theta)\right)\\
&-2R\int_0^{2\pi}\sin\theta\sqrt{R^2(1+\sin\theta)^2+z_2^2}\tan^{-1}\left[\frac{x_{2,i}-R\cos\theta}{\sqrt{R^2(1+\sin\theta)^2+z_2^2}}\right]\mathrm{d}\theta
\end{aligned} \tag{6b}$$

in which $a=x_{2,i}^2+z_2^2+2R^2$; $b=-2Rx_{2,i}$; and $c=2R^2$. The lower and upper limits of integration on contour C_1 are $\theta=0$ and $\theta=2\pi$, respectively. On contour C_2, $j=1,2$ denotes lower and upper limits of integration, respectively, which corresponds to endpoint coordinates.

Alternatively, a correlation is developed for the specific case of a view factor between a disk of unit radius and a rectangle placed symmetric about the y–z plane, with one of its edges coinciding with the plane containing the disk.

The correlation, for the range $0.1\le L\le 10.0$ and $0.1\le B\le 10.0$,

$$F_{2\text{–}1}=\frac{0.5974[(0.05L+L^2)^{0.4976}(0.05B+B^2)^{0.2091}+0.5974(0.1A_2+A_2^2)^{0.0613}]}{(L+L^4)^{0.4685}+(B+B^4)^{0.30775}+(A_2+A_2^2)^{0.7457}} \tag{7}$$

is based on 7858 data points. The maximum error is ±19.88% and the correlation coefficient is 0.997. Of these many data points, only 526 data points have an error of greater than ±10%.

The view factor between the disk and the rectangle, when the rectangle is placed arbitrarily on the x–z plane with its edges being either parallel or perpendicular to the plane of the disk, can be calculated using the correlation presented and view factor algebra.

Case 3 View factor between two rectangles in perpendicular planes: Consider the view factor between rectangles on perpendicular planes as shown in Fig. 2, when one rectangle is placed on the plane containing another rectangle, which is placed arbitrarily. The coordinate is chosen such that the rectangle 1lies on the x–y plane and rectangle 2 lies on the x–z plane. The corner points of rectangle 1 are denoted as $P_{1,i}$ and of rectangle 2 by $P_{2,i}$, along the direction of integration. Since $z_1=0$ and $y_2=0$, integrations along $\mathrm{d}y_1\,\mathrm{d}y_2$ and $\mathrm{d}z_1\,\mathrm{d}z_2$ vanish. Thus Eq. (1) reduces to

$$F_{1\text{–}2}=\frac{1}{4\pi A_1}\oint_{C_1}\oint_{C_2}\ln\left[(x_2-x_1)^2+y_1^2+z_2^2\right]\mathrm{d}x_1\,\mathrm{d}x_2 \tag{8}$$

On contour C_1, x_1 remains constant along the paths $P_{1,2}$–$P_{1,3}$ and $P_{1,4}$–$P_{1,1}$, and hence the integrations along $\mathrm{d}x_1\,\mathrm{d}x_2$ vanish. Further, y_1 remains constant along the paths $P_{1,1}$–$P_{1,2}$ and $P_{1,3}$–$P_{1,4}$, while x_1 varies. On contour C_2, z_2 is represented in terms of x_2 using equation of straight line $z_2=m_2x_2+c_2$, and the closed form solution (to the possible extent) of Eq. (8) along the said paths can be shown to be

$$F_{1\text{–}2}=\frac{1}{4\pi A_1}\sum_{i=1,3}^{2,4}\sum_{j=1}^{2}\sum_{k=1}^{4}\sum_{l=1}^{2}(-1)^{j+l}G(x_{1,i},y_1,x_{2,k},z_{2,k}) \tag{9a}$$

where

$$\begin{aligned}
G(x_{1,i},y_1,x_{2,k},z_{2,k})={}&\frac{1}{2M_2}\left\{2\left[(M_2x_{2,k}+m_2c_2)(x_{1,i}-x_{2,k})\right.\right.\\
&\left.\left.+x_{2,k}^2\right]-\left(x_{2,k}^2+y_1^2+z_{2,k}^2+x_{1,i}^2\right)\right\}\ln\left[(x_{2,k}-x_{1,i})^2+y_1^2+z_{2,k}^2\right]\\
&+\frac{2[x_{2,k}(M_2-1)+m_2c_2]\sqrt{z_{2,k}^2+y_1^2}}{M_2}\tan^{-1}\left(\frac{x_{1,i}-x_{2,k}}{\sqrt{z_{2,k}^2+y_1^2}}\right)\\
&+\frac{x_{1,i}\{x_{1,i}+2[x_{2,k}(1-4M_2)-2m_2c_2]\}}{2M_2}\\
&+\frac{2}{M_2}\sum_{k=1}^{4}\sum_{l=1}^{2}(-1)^l\int_{C_1}\sqrt{M_2y_1^2+(c_2+m_2x_1)^2}\tan^{-1}\\
&\times\left[\frac{M_2x_{2,k}+m_2c_2-x_1}{\sqrt{M_2y_1^2+(z_{2,k}+m_2x_1)^2}}\right]\mathrm{d}x_1
\end{aligned} \tag{9b}$$

in which $M_2=1+m_2^2$. Here j and l denote the limits of integrations on contours C_1 and C_2, respectively. When all the edges of rectangle 2 are placed either parallel or perpendicular to the plane containing rectangle 1, then Eq. (9a) reduces to configuration C-15 available in the literature [1].

Alternatively, correlations are developed for a specific case of view factor between a unit square and a square of side S which is placed arbitrarily on a perpendicular plane with one of its corners touching the plane containing a unit square. Two different correlations are presented below to minimize the maximum error in

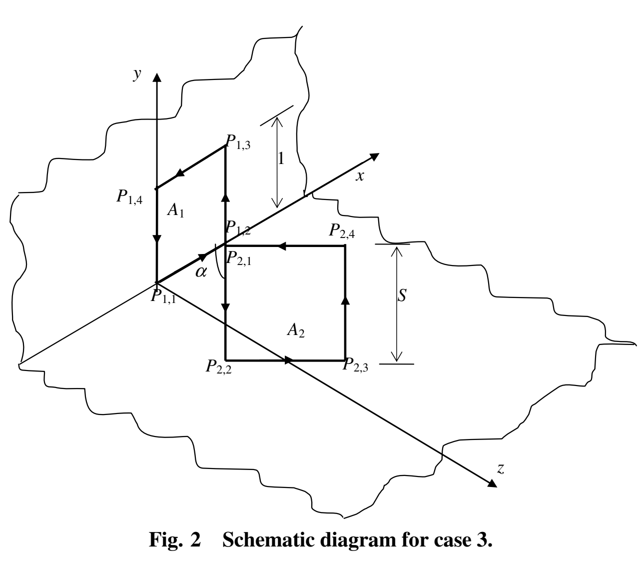

Fig. 2 Schematic diagram for case 3.

the predicted view factor: The correlation for the range $0 \leq \alpha \leq \pi/2$ and $0.1 \leq S \leq 2.0$,

$$F_{1-2} = \frac{0.4399 S^{0.0871}}{\alpha^{0.0187} + (S^{-2.0849} + \alpha^{0.0871})^{0.6387} (S^{1.5473} + \alpha^{1.4836})^{1.5932}} \tag{10a}$$

is based on 460 data points. The maximum error is $\pm 9.08\%$, and the correlation coefficient is 0.999.

The correlation for the range $0 \leq \alpha \leq \pi/2$ and $2.0 \leq S \leq 10.0$,

$$F_{1-2} = \frac{0.2577 S^{-1.6494} \alpha^{-1.8087}}{S^{0.059} \alpha^{-0.3392} + (1 + S\alpha^{-8.3298})^{0.2152}} \tag{10b}$$

is based on 368 data points. The maximum error is $\pm 12.25\%$, and the correlation coefficient is 0.999.

The view factor between squares placed arbitrarily on perpendicular planes can be calculated using the correlations presented and view factor algebra.

Conclusion

Expressions for the view factor between three pairs of configurations, a disk and a rectangle on parallel planes, a disk and a rectangle on perpendicular planes, and two rectangles on perpendicular planes, are presented. Closed form solutions have been developed to the extent possible, in which appropriate limits have to be substituted to determine the view factor. Alternatively, correlations have been presented for three specific configurations, after evaluating the nonintegrable terms using Simpson's one-third rule. However, view factors for general configurations could be calculated using the correlations presented and view factor algebra. For the rectangles in perpendicular planes, when the edges of rectangles are either parallel or perpendicular, the closed form solution reduces to the corresponding particular case available in the literature.

Acknowledgment

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